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A Study on Discrete Model of a Typical Three Species Syn-Ecology with Limited Resources

B. Hari Prasad*

Department of Mathematics, Chaitanya Degree College (Autonomous), Hanamkonda, Telangana State, India

Abstract

In this paper, we study on a three species discrete model syn ecology with limited resources. The system comprises of a commensal (S1), two hosts S2 and S3ie., S2 and S3 both benefit S1, without getting themselves effected either positively or adversely. Further S2 is a commensal of S3 and S3 is a host of both S1, S2. The basic equations for this model constitute as three first order non-linear coupled ordinary difference equations. All possible equilibrium points are identified based on the model equations and criteria for their stability are discussed. The model would be stable if absolute value of each of the eigen values of the characteristic equation is less than one.

Keywords

Commensal, Eigen Value, Equilibrium Point, Host, Stable, Unstable

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1. Introduction

Ecology, a branch evolutionary biology, deals with living species that coexist in a physical environment that sustain themselves on common resources. It is a common observation that the species of same nature can not flourish in isolation without any interaction with species of different kinds. Syn-ecology is an ecosystem comprised of two or more distinct species. Species interact with each other in one way or other. The Ecological interactions can be broadly classified as Ammensalism, Competition, Commensalism, Neutralism, Mutualism, Predation, Parasitism and so on. Lotka[1], Svirezhev et al [2] and Volterra [3] pioneered theoretical ecology significantly and opened new eras in the field of life and biological sciences. The authors Rogers et al [4], Varma [5] and Veilleux [6] discussed prey-predator, competing ecological models. Colinvaux [7] and Smith [8] studied basic concepts of population models in ecology.

Mathematical Modeling plays a vital role in providing insight into the mutual relationships (positive, negative) between the interacting species. The general concepts of modeling have been discussed by several authors Kapur [9], Kushing [10], Meyer [11] and Pielou [12]. Srinivas [13] studied the competitive ecosystem of two species and three species with limited and unlimited resources. Later, Narayan et al [14]studied prey-predator ecological models with partial cover for the prey and alternate food for the predator. Acharyulu et al [15, 16] derived some productive results on various mathematical models of ecological Ammensalism with multifarious resources in the manifold directions. Further, Kumar [17] studied some mathematical models of ecological commensalism. The present author Prasad [18-21] investigated continuous and discrete models on the three species syn-ecosystems.

The present investigation is on an analytical study of a typical three species (S_1, S_2, S_3) syn-eco system. The system comprises of a commensal (S_1), two hosts S_2 and S_3ie, S_2 and S_3 both benefit S_1, without getting themselves effected either positively or adversely. Further S_2 is a commensal of S_3 and S_3 is a host of both S_1, S_2. Commensalism is a symbiotic interaction between two populations where one population (S_1) gets benefit from (S_2) while the other (S_2) is neither

* Corresponding author
E-mail address: sumathi_prasad73@yahoo.com

harmed nor benefited due to the interaction with (S_1). The benefited species (S_1) is called the commensal and the other, the helping one (S_2) is called the host species. A common example is an animal using a tree for shelter-tree (host) does not get any benefit from the animal (commensal).Some more real-life examples of commensalism are, i. The clownfish shelters among the tentacles of the sea anemone, while the sea anemone is not effected. ii. A flatworm attached to the horse crab and eating the crab's food, while the crab is not put to any disadvantage. iii. Sucker fish (echeneis) gets attached to the under surface of sharks by its sucker. This provides easy transport for new feeding grounds and also food pieces falling from the sharks prey, to Echeneis.

2. Material and Methods

2.1. Notation Adopted

S_1: Commensal of S_2 andS_3;S_2: Host of S_1 and commensal of S_3;S_3: Host of S_1 and S_2; $N_i(t)$: The population strength of S_i at time t, i = 1,2,3;t: Time instant; a_i: Natural growth rate of S_i , i = 1,2, 3;a_{ii}:Self inhibition coefficients of S_i, i=1, 2, 3;a_{12},a_{13}: Interaction coefficients of S_1 due to S_2 and S_1 due to S_3;a_{23}: Interaction coefficient of S_2 due to S_3.Further the variables N_1, N_2, N_3 are non-negative and the model parameters a_1, a_2, a_3, a_{11}, a_{22}, a_{33}, a_{13}, a_{23} are assumed to be non-negative constants.

2.2. Theoretical Framework: Basic Equations

Consider the non-linear autonomous system of discrete difference equations

$$N_1(t)-N_1(t-1)=a_1N_1(t-1)-a_{11}N_1^2(t-1)+a_{12}N_1(t-1)N_2(t-1)+a_{13}N_1(t-1)N_3(t-1) \quad (1)$$

$$N_2(t)-N_2(t-1)=a_2N_2(t-1)-a_{22}N_2^2(t-1)+a_{23}N_2(t-1)N_3(t-1) \quad (2)$$

$$N_3(t)-N_3(t-1)=a_3N_3(t-1)-a_{33}N_3^2(t-1) \quad (3)$$

The equations (1),(2) and (3) can be written in terms of recurrence relations as

$$N_1(t)=g_1(N_1,N_2,N_3)=\alpha_1N_1(t-1)-a_{11}N_1^2(t-1)+a_{12}N_1(t-1)N_2(t-1)+a_{13}N_1(t-1)N_3(t-1) \quad (4)$$

$$N_2(t)=g_2(N_1,N_2,N_3)=\alpha_2N_2(t-1)-a_{22}N_2^2(t-1)+a_{23}N_2(t-1)N_3(t-1) \quad (5)$$

$$N_3(t)=g_3(N_1,N_2,N_3)=\alpha_3N_3(t-1)-a_{33}N_3^2(t-1) \quad (6)$$

where $\alpha_i=(a_i+1)\geq 1$, $i=1,2,3$

The equilibrium states for the given system are obtained by solving the equations at

$$N_i(t+1)=N_i(t) \ , \ i=1,2,3$$

We get,

$$E_1:\bar{N}_1=0,\bar{N}_2=0,\bar{N}_3=0$$

$$E_2:\bar{N}_1=0,\bar{N}_2=0,\bar{N}_3=\frac{\alpha_3-1}{a_{33}}$$

$$E_3:\bar{N}_1=0,\bar{N}_2=\frac{\alpha_2-1}{a_{22}},\bar{N}_3=0$$

$$E_4:\bar{N}_1=\frac{\alpha_1-1}{a_{11}},\bar{N}_2=0,\bar{N}_3=0$$

$$E_5:\bar{N}_1=0,\bar{N}_2=\beta_2,\bar{N}_3=\frac{\alpha_3-1}{a_{33}}$$

where

$$\beta_2=\frac{(\alpha_2-1)a_{33}+(\alpha_3-1)a_{23}}{a_{22}a_{33}}$$

$$E_6:\bar{N}_1=\beta_1,\bar{N}_2=0,\bar{N}_3=\frac{\alpha_3-1}{a_{33}}$$

where

$$\beta_1=\frac{(\alpha_1-1)a_{33}+(\alpha_3-1)a_{13}}{a_{11}a_{33}}$$

$$E_7:\bar{N}_1=\gamma_1,\bar{N}_2=\frac{\alpha_2-1}{a_{22}},\bar{N}_3=0$$

where

$$\gamma_1=\frac{(\alpha_1-1)a_{22}+(\alpha_2-1)a_{12}}{a_{11}a_{22}}$$

$$E_8:\bar{N}_1=\frac{\alpha_1-1}{a_{11}}+\frac{a_{12}\beta_2}{a_{11}}+\frac{a_{13}(\alpha_3-1)}{a_{11}a_{33}},\ \bar{N}_2=\beta_2,\bar{N}_3=\frac{\alpha_3-1}{a_{33}}$$

2.3. Stability of the Equilibrium States

The basic equations can be linearized about the equilibrium point $E(\bar{N}_1,\bar{N}_2,\bar{N}_3)$.

We get,

$$g_i(N_1,N_2,N_3) = g_i(\bar{N}_1,\bar{N}_2,\bar{N}_3) + \frac{\partial\,[g_i(\bar{N}_1,\bar{N}_2,\bar{N}_3)]}{\partial N_1}(N_1-\bar{N}_1) + \frac{\partial\,[g_i(\bar{N}_1,\bar{N}_2,\bar{N}_3)]}{\partial N_2}(N_2-\bar{N}_2) + \frac{\partial\,[g_i(\bar{N}_1,\bar{N}_2,\bar{N}_3)]}{\partial N_3}(N_3-\bar{N}_3)$$

where $i = 1,2,3$

$$N_1(t) = \left(\alpha_1 - 2a_{11}\bar{N}_1 + a_{12}\bar{N}_2 + a_{13}\bar{N}_3\right)N_1 + a_{12}\bar{N}_1N_2 + a_{13}\bar{N}_1N_3 \tag{7}$$

$$N_2(t) = \left(\alpha_2 - 2a_{22}\bar{N}_2 + a_{23}\bar{N}_3\right)N_2 + a_{23}\bar{N}_2N_3 \tag{8}$$

$$N_3(t) = \left(\alpha_3 - 2a_{33}\bar{N}_3\right)N_3 \tag{9}$$

The above equations can be written in the matrix notation as,

$$N(t) = A.N(t-1)\ .$$

where

$$A(E) = \begin{bmatrix} \alpha_1 - 2a_{11}\bar{N}_1 + a_{12}\bar{N}_2 + a_{13}\bar{N}_3 & a_{12}\bar{N}_1 & a_{13}\bar{N}_1 \\ 0 & \alpha_2 - 2a_{22}\bar{N}_2 + a_{23}\bar{N}_2 & a_{23}\bar{N}_2 \\ 0 & 0 & \alpha_3 - 2a_{33}\bar{N}_3 \end{bmatrix} \tag{10}$$

and $N = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix}$

The characteristic equation for the system is

$$\det[A - \lambda I] = 0 \tag{11}$$

The equilibrium point $E(\bar{N}_1,\bar{N}_2,\bar{N}_3)$ is stable, if absolute value of each of the eigen values of the characteristic equation (11) is less than one.

2.3.1. Stability of E_1

In this case, we get,

$$A(E_1) = \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}$$

The eigen values of the matrix $A(E_1)$are α_1, α_2, α_3.Since, the absolute values of these three are greater than or is equal to one. i.e,$|\alpha_1|,|\alpha_2|,|\alpha_3| \geq 1$.

Hence, E_1 (0, 0) is unstable.

2.3.2. Stability of E_2

In this case, we get,

$$A(E_2) = \begin{bmatrix} \alpha_1 + \dfrac{a_{13}(\alpha_3-1)}{a_{33}} & 0 & 0 \\ 0 & \alpha_2 + \dfrac{a_{23}(\alpha_3-1)}{a_{33}} & 0 \\ 0 & 0 & 2-\alpha_3 \end{bmatrix}$$

The eigen values of the matrix $A(E_2)$are $\left(\alpha_1 + \dfrac{a_{13}(\alpha_3-1)}{a_{33}}\right), \left(\alpha_2 + \dfrac{a_{23}(\alpha_3-1)}{a_{33}}\right), 2-\alpha_3$.

Since, the absolute value of all these eigen values is less than one only when

$$-1 < \left[\left(\alpha_1 + \frac{a_{13}\alpha_3}{a_{33}}\right) - \frac{a_{13}}{a_{33}}\right] < 1,$$

$$-1 < \left[\left(\alpha_2 + \frac{a_{23}\alpha_3}{a_{33}}\right) - \frac{a_{23}}{a_{33}}\right] < 1, 1 < \alpha_3 < 3$$

Hence, E_2 is stable.

2.3.3. Stability of E_3

In this state, we have,

$$A(E_3) = \begin{bmatrix} \alpha_1 + \dfrac{a_{12}(\alpha_2-1)}{a_{22}} & 0 & 0 \\ 0 & 2-\alpha_2 & \dfrac{a_{23}(\alpha_2-1)}{a_{22}} \\ 0 & 0 & \alpha_3 \end{bmatrix}$$

The eigen values of the matrix $A(E_3)$are $\left(\alpha_1 + \dfrac{a_{12}(\alpha_2-1)}{a_{22}}\right), 2-\alpha_2 \ and \ \alpha_3$.

Since, $|\alpha_3| \geq 1$ .Hence, E_3 is unstable.

2.3.4. Stability of E_4

In this state, we have,

$$A(E_4) = \begin{bmatrix} 2-\alpha_1 & \dfrac{a_{12}(\alpha_1-1)}{a_{11}} & \dfrac{a_{13}(\alpha_1-1)}{a_{11}} \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}$$

The eigen values of the matrix A(E_4) are $2-\alpha_1, \alpha_2, \alpha_3$.

Since, $|\alpha_2|, |\alpha_3| \geq 1$. Hence, E_4 is unstable.

2.3.5. Stability of E_5

In this case,

$$A(E_5)=\begin{bmatrix} \alpha_1 + a_{12}\beta_2 + \frac{a_{13}(\alpha_3-1)}{a_{33}} & 0 & 0 \\ 0 & 2-\alpha_2-\frac{a_{23}(\alpha_3-1)}{a_{33}} & a_{23}\beta_2 \\ 0 & 0 & 2-\alpha_3 \end{bmatrix}$$

The eigen values of the matrix A(E_5)are $\left(\alpha_1 + a_{12}\beta_2 + \frac{a_{13}(\alpha_3-1)}{a_{33}}\right)$, $\left(2-\alpha_2-\frac{a_{23}(\alpha_3-1)}{a_{33}}\right), 2-\alpha_3$. Since, the absolute value of all these eigen values is less than one only when

$$-1<\left[\left(\alpha_1 + a_{12}\beta_2 + \frac{a_{13}\alpha_3}{a_{33}}\right)-\frac{a_{13}}{a_{33}}\right]<1,$$

$$-1<\left[\left(2+\frac{a_{23}}{a_{33}}\right)-\left(\alpha_2+\frac{a_{23}\alpha_3}{a_{33}}\right)\right]<1, 1<\alpha_3<3$$

Hence, E_5 is stable.

2.3.6. Stability of E_6

In this case,

$$A(E_6)=\begin{bmatrix} 2-\alpha_1-\frac{a_{13}(\alpha_3-1)}{a_{33}} & a_{12}\beta_1 & a_{13}\beta_1 \\ 0 & \alpha_2+\frac{a_{23}(\alpha_3-1)}{a_{33}} & 0 \\ 0 & 0 & 2-\alpha_3 \end{bmatrix}$$

The eigen values of the matrix A(E_6)are $\left(2-\alpha_1-\frac{a_{13}(\alpha_3-1)}{a_{33}}\right), \left(\alpha_2+\frac{a_{23}(\alpha_3-1)}{a_{33}}\right), 2-\alpha_3$

Since, the absolute value of all these eigen values is less than one only when

$$-1<\left[\left(2+\frac{a_{13}}{a_{33}}\right)-\left(\alpha_1+\frac{a_{13}\alpha_3}{a_{33}}\right)\right]<1,$$

$$-1<\left[\left(\alpha_1+\frac{a_{23}\alpha_3}{a_{33}}\right)-\frac{a_{23}}{a_{33}}\right]<1, \ 1<\alpha_3<3$$

Hence, E_6 is stable.

2.3.7. Stability of E_7

In this state,

$$A(E_7)=\begin{bmatrix} 2-\alpha_1-\frac{a_{12}(\alpha_2-1)}{a_{22}} & a_{12}\gamma_1 & a_{13}\gamma_1 \\ 0 & 2-\alpha_2 & \frac{a_{23}(\alpha_2-1)}{a_{22}} \\ 0 & 0 & \alpha_3 \end{bmatrix}$$

The eigen values of the matrix A(E_7)are $\left(2-\alpha_1-\frac{a_{12}(\alpha_2-1)}{a_{22}}\right), \ 2-\alpha_2, \ \alpha_3$

Since, $|\alpha_3| \geq 1$. Hence, E_7 is unstable.

2.3.8. Stability of E_8

In this state,

$$A(E_8)=\begin{bmatrix} 2-\alpha_1-\delta_1 & a_{12}\bar{N}_1 & a_{13}\bar{N}_1 \\ 0 & 2-\alpha_2-\frac{a_{23}(\alpha_3-1)}{a_{33}} & a_{23}\beta_2 \\ 0 & 0 & 2-\alpha_3 \end{bmatrix}$$

The eigen values of the matrix A(E_8)are $2-\alpha_1-\delta_1, \left(2-\alpha_2-\frac{a_{23}(\alpha_3-1)}{a_{33}}\right), \ 2-\alpha_3$

where $\delta_1 = a_{12}\beta_2 + \frac{a_{13}(\alpha_3-1)}{a_{33}}$

Since, the absolute value of all these eigen values is less than one only when

$$1<\alpha_1+\delta_1<3, \ 1<\alpha_3<3,$$

$$-1<\left[\left(2+\frac{a_{23}}{a_{33}}\right)-\left(\alpha_2+\frac{a_{23}\alpha_3}{a_{33}}\right)\right]<1$$

Hence, E_8 is stable.

3. Discussion

Investigate some relation-chains between the species such as Prey-Predation, Neutralism, Commensalism, Mutualism, Competition and Ammensalism between three species (S_1, S_2, S_3) with the population relations.

The present paper deals with an investigation on a typical three species discrete model syn eco-system. The system comprises of a commensal (S_1), two hosts S_2 and S_3ie., S_2

and S_3 both benefit S_1, without getting themselves effected either positively or adversely. Further S_2 is a commensal of S_3 and S_3 is a host of both S_1, S_2. It is observed that, in all eight equilibrium states, only the four states E_2,E_5,E_6,E_8 are stable and rest of them unstable.

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